The Dynamic Properties of a FOUR-DIMENSIONAL, Complex Manifold With A Uniform Acceleration FIeld

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# Abstract

Friedman’s solutions to Einstein’s field equations do not make accurate predictions about the universe on the largest scales. Even with multiple free parameters, or perhaps because of them, General Relativity gives conflicting answers when asked for the rate of expansion of the universe. We offer as an alternative a model of expansion employing a four-dimensional, complex manifold with one imaginary dimension of time and three real dimensions of space and no other gimmicks. Like all manifolds, this one has an acceleration field. Unlike other manifolds, this field is not assumed to be zero. Here we demonstrate that General Relativity, with a model of uniform acceleration in four dimensions, makes more accurate predictions on all scales than it does with an assumption of linear time. When given the proper metric tensor, General Relativity can even predict the Tully-Fisher Relationship. However, to do this, we must abandon one of our most sacred beliefs: that objects at rest remain at rest.

# Introduction

With every new revelation from our space telescopes, the *Theory of General Relativity* appears to be more broken. When the field equations are solved using the FLRW metric tensor, they give conflicting answers for the rate of expansion of the universe () that is beyond the ability of the experimenters to explain (Krishnan et al. 2021). It gives one answer when we look at background radiation, and a different answer when we look at supernovae. In the language of the Scientific Method, the Friedman solutions to the Einstein Field Equations have mutually disproven themselves.

Perhaps there is more new physics to be discovered? General Relativity already depends on multiple ad hoc theories – essentially free parameters in the energy-momentum density term – that have no plausible theoretical foundation (e.g., Dark Energy). Expecting different results with more new physics would be irrational.

Perhaps an assumption was missed? Yes, perhaps. Shall we start with the first one? All forces cause acceleration. Not all acceleration is caused by forces. Some acceleration is caused by curvature. The curvature of coordinate time with respect to proper time, for example, would be observed as an acceleration without a force. There is no geometrical foundation for assuming that two particles would not drift apart when all the forces are removed. If the curvature is not known, then the acceleration is not known. What law allows us to assume that objects at rest remain at rest?

There is none. Objects in freefall will follow the surface of the manifold. That is the only law that matters. If that surface possesses an acceleration, then free-falling objects on that surface will also accelerate as described by:

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Where is the acceleration of a point on the manifold, is the velocity, is the evolution parameter (proper time), and represents the basis vectors for the four dimensions – time (, radius (), polar angle (), azimuthal angle () – in this coordinate system where . This relation can be expanded with the chain rule:

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Where is the proper acceleration of an object, and is the acceleration from the basis vectors changing with respect to the coordinates. Rearranging, the formula for the proper acceleration of a test particle on the surface of this manifold having a non-zero acceleration field, , is:

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# Galaxy Rotation Curves

In non-relativistic domains, the equation describing the path of this test particle in the presence of a collection of fictitious forces, , is:

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Where is the momentum. Replacing the general terms of this equation with specific terms for centripetal motion in a gravitational pseudo-force field yields the formula for motion.

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Where is the gravitational constant, is the mass within a radius, , and is the tangential velocity of an object at the given radius. What, then, would freefall motion look like on the surface of complex manifold with a uniform acceleration field?

A graph of a graph

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Figure 1 - The rotation curves of orbital motion around a spherical mass of having a radius of and constant density, for a variety of freefall accelerations (in units of ).

Figure 1 demonstrates that the greater the acceleration field of the manifold, the faster an object in that field will orbit a given mass. This is an easily identified property that sets quadratic expansion apart from flat spacetime. At higher values of , we see a flattening of the rotation curves on a galactic scale, and at , we recover Newtonian physics.

# The Tully-FisheR Relation

Orbital systems in a uniform acceleration field possess a plane of mass that is a function of radius and tangential velocity. Eq. (2) can be rearranged to predict the enclosed mass in such a system.

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A graph of a graph showing a curve

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Figure 2 - The fundamental plane of mass, in units of , for , as a function of radius, in , and velocity, in .

This relationship suggests that there is a radius beyond which the acceleration field outward overcomes the gravitational field inward, as seen in Figure 2, placing an upper bound on the amount of mass that can be enclosed in an orbit. The radius of this inflection point is:

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Substituting Eq. (5) back into Eq. (4) yields the formula for the maximum mass, , as a function of tangential velocity:

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A study of the relationship between velocity and baryonic mass was conducted in (McGaugh 2012). Using the data from this study and employing a minimization algorithm on Eq. (6), we find a value of for , thus:

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Where velocity has units of and mass has units of .

Blue dots on a black background

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Figure 3 - The relationship between tangential velocity and baryonic mass. The blue circles are the combined gas and stellar mass of gas-rich galaxies and the solid line is the maximum mass possible in quadratically expanding spacetime.

Eq. (7) predicts the Tully-Fisher relation with one free parameter. Quadratic expansion predicts that galaxies will have a brightness that is limited by the fourth power of their orbital speed. This is another easily identified property of a manifold with a uniform acceleration field. The FLRW metric, employing an assumption of Lorentz symmetry, does not predict such a relation.

# The Metric Formula

The distance between two points, , on this manifold is described by:

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where is the metric tensor, and is the difference between two coordinates in each dimension. If we consider only the time and the radius dimensions, and assume their coordinate axes are orthogonal, this formula resolves to:

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where , is the change in proper distance and is the change in proper time. Observers can only measure coordinate distances, so a conversion between proper and coordinate distance is needed. The coordinate distance can be found by taking the second integral of the uniform acceleration. Note that time on this manifold is imaginary, so we integrate by substitution, and that the displacement formulas are the same for all dimensions, so we drop the index.

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Where is the tangent velocity of the manifold at time, , and the constant of integration is . Eq. (11) describes uniform acceleration in four dimensions.

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Figure 4 – The spacetime interval, , from to showing the line elements space () and time ().

The imaginary temporal line element, , over the interval to can be expressed as:

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The real spatial line element, , is found by describing the relationship between and as a ratio of the displacement to the size of the manifold, , over the same interval.

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Where is the manifold size at and is the manifold size at . The line element, , is the coordinate distance, , minus one half of expansion, expressed as :

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The metric formula for a two-dimensional distance on this manifold using practical coordinates is:

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# Type Ia Supernovae

Three initial conditions (free parameters) are needed to describe this manifold. The acceleration, , the initial tangent velocity, , and the age of the manifold, , must be discovered empirically. The acceleration has already been established from the Tully-Fisher relation. The initial tangent velocity, , can be derived from the measured speed of light if we assume that light (causality) travels at the tangent velocity of the manifold. From Eq. (10), we have:

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The age of the universe can be found using a special kind of supernovae, type Ia, that explodes with a predictable luminosity. The distance can be found by solving Eq. (15) for :

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This formula can be simplified by encoding the time coordinates as:

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Where is the spatial distance as a function of redshift and is the redshift. This redshift value is encoded in photons making it possible to use a volume of photons from a known source of luminosity as distance markers. As a function of redshift, the luminous distance, , is:

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A screen shot of a graph

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Figure 5 - Top: The luminous distances to a selection of 1071 Ia supernovae (green) and the distance predicted by luminous distance formula in Eq. (18) (red) and, for comparison, the distance predicted by the FLRW formula (blue) using the parameters from (Planck Collaboration et al. 2020). Bottom: The difference between the predicted luminous distance and the observed value in magnitude.

The age of the universe can be extracted from the Pantheon data set (Scolnic et al. 2022) using a minimization algorithm on Eq. (18). The reduced of for uniform acceleration with just one free parameter (age) is a better match to the observed data than for FLRW with its four free parameters (inflation density, matter density, curvature density and cosmological constant).

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Table 1 – The initial conditions: the uniform acceleration, the initial tangent velocity, and the age of the manifold.

The SNe Ia data describes a manifold that is 13.6 billion years old and currently has a tangent velocity of . Using this metric, a galaxy with a redshift of 12 would be one billion years old. Using FLRW, the same galaxy would be 360 million years old.

# the Einstein Field Equations

To solve the Einstein Field Equations, the displacements of Eq. (15) are converted to infinitesimals by reversing the integration and assuming that all measurements of start at .

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Gathering the terms and adding the polar and azimuthal dimensions, metric tensor for this manifold is:

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The non-zero Christoffel Symbols are enumerated according to the formula

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The Riemann Tensor is expanded using the formula:

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Yielding the Ricci Tensor :

Which, in turn, yields the Ricci Scalar

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We are generally working towards a formula where we can equate the curvature of spacetime to the contents. The contents of the manifold are modelled as a perfect fluid that moves only through time.

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The trace of this tensor is:

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With these terms in hand, the solutions to the EFEs can now be calculated.

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These solutions yield predictions for the pressure of the manifold, , and the baryonic density as a function of curvature, .

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# Curvature

Fluctuations in one or more quantum fields at the dawn of time resulted in tiny density differences which, in turn, set up standing sound waves that collapsed when the plasma cooled enough to allow photons to stream freely. Those photons carried away with them an image of the temperature differences on the surface of the manifold at the time of the wave collapse. This light is observed today as Cosmic Microwave Background (CMB) Radiation, and it contains information about the curvature of spacetime.

# Data Availibility

The data and Mathematica notebooks underlying this article are available at https://github.com/DonaldAirey/quadratically-expanding-space.

# Bibliography

Krishnan, C., Mohayaee, R., Colgáin, E. Ó., Sheikh-Jabbari, M. M., & Yin, L. 2021, Class Quantum Gravity, 38, 184001

McGaugh, S. S. 2012, Astron J, 143, 40

Planck Collaboration, Aghanim, N., Akrami, Y., et al. 2020, Astron Astrophys, 641, A6

Scolnic, D., Brout, D., Carr, A., et al. 2022, Astrophys J, 938, 113

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