The Dynamic Properties of a FOUR-DIMENSIONAL, pseudo-Riemannian Manifold With Uniform Acceleration

Donald Airey[[1]](#footnote-1)

# Abstract

Friedman’s solutions to Einstein’s field equations do not make accurate predictions about the universe on the largest scales. Even with multiple free parameters (Dark Matter, Inflation, Dark Energy), General Relativity gives conflicting answers for the rate of expansion of the universe. Here we offer an alternative model of expansion employing a four-dimensional, pseudo-Riemannian manifold with one imaginary dimension of time and three real dimensions of space and no other gimmicks. Like all manifolds, this one has a ubiquitous acceleration tensor. Unlike other manifolds, this tensor is not assumed to be zero. We then provide an evolution parameter, , and describe the resulting dynamic properties. Here we demonstrate that General Relativity, with an expansion model based on constant acceleration, makes more accurate predictions on all scales than FLRW, with an expansion model based on unproven physics. When given the proper metric tensor, General Relativity can even predict the Tully-Fisher Relationship. However, to do this, we must abandon one of our most sacred beliefs: that objects at rest remain at rest.

# Introduction

With every new revelation from our space telescopes, the *Theory of General Relativity* appears to be more broken. When the field equations are solved using the FLRW metric, they give conflicting answers for the rate of expansion of the universe () that is beyond the ability of the experimenters to explain. In the language of the Scientific Method, the Friedman solutions to the Einstein Field Equations have disproven themselves.

Perhaps there is more new physics to be discovered? General Relativity already depends on multiple ad hoc theories that have no plausible theoretical foundation. To paraphrase the author of this theory, expecting a different result with more new physics is insane.

Perhaps an assumption was missed? Yes, perhaps. Shall we start with the very first one? All forces cause acceleration. Not all acceleration is caused by forces. Some acceleration is caused by curvature. The curvature of coordinate time with respect to proper time, for example, would be observed as an acceleration without a force. It is illogical to assume that there would be no acceleration in your thought experiment were you to remove all forces. What law of logic or geometry allows you to assume that objects at rest remain at rest?

There is none. Objects in freefall will follow the surface of the manifold. That is the only law that matters. If that surface possesses an acceleration, then free-falling objects on that surface will also accelerate.

Substituting a variable for an assumption, freefall motion is more generally described by:

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Where is the acceleration tensor of the manifold, is the velocity, is the evolution parameter (proper time), and represents the basis vectors. This relation can be expanded with the chain rule:

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Where is the proper acceleration of an object, and is the acceleration caused by the quadratic change in the basis vectors with time. Rearranging, the formula for the proper acceleration of a test particle on the surface of this manifold having a ubiquitous acceleration tensor, , is:

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In non-relativistic domains, the equation describing the path of this test particle in the presence of a collection of fictitious forces, , is:

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Where is the momentum tensor. We can derive a formula for orbital motion in a gravitational field by replacing the general terms of this equation with specific terms:

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Where is the gravitational constant, is the mass within a radius, , and is the tangential velocity of an object at the given radius. What, then, would freefall motion look like on the surface of pseudo-Riemannian manifold with a ubiquitous acceleration of ?

Figure 1 - This figure shows the rotation curves of orbital motion around a spherical mass of having a radius of and constant density, for a variety of freefall accelerations (in units of ). Note that at a value of , we recover Newtonian Mechanics. At higher values of , we see a flattening of the rotation curves on a galactic scale.

# The Mechanics of Expansion

We can construct a metric formula by substituting Eq. **Error! Reference source not found.** into Eq. **Error! Reference source not found.** for the blue space term, , and then deriving an expression for the magenta space term, .

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Where is the time of an observation, and is the arbitrary time of some event. However, we have no way to directly measure in expanding space. The schematic of Figure 2 illustrates that the only values that can be measured directly are and . We need a method to relate the coordinate distance, , to the line element, .

A picture containing text, clock

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Figure 2 - The interval, , from to in quadratically expanding space showing the line elements, and .

Using Eq. **Error! Reference source not found.**, the relation between and , as the manifold expands, is:

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Where is the spatial extent at and is the spatial extent at . Next, we will observe that the line element, , is the coordinate distance, , less one half of expansion (expressed as ):

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The metric formula for a two-dimensional distance using practical coordinates is then:

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# Four-Dimensional Metric Formula

We now consider the permutations of space and time. Two dimensions of time result in only one spatial dimension, so any discussion would be of limited value. Three dimensions of time result in three dimensions of space, one for each imaginary plane. This option agrees with the observed inverse-square law, so let us put a pin in it. If we had four dimensions of time, then we would observe six dimensions of space. We do not, so this option also shows little promise.

Having considered the permutations, we will focus the discussion on a manifold with three imaginary dimensions of time and three real dimensions of space. We are going to label them as red time, , green time, and blue time, due to the way they produce secondary dimensions. The product of blue time and green time is cyan space, . The product of green time and red time is yellow space, . The additional line elements are:

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These line elements combine to give us the metric formula for the six-dimensional distance:

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We can simplify this formula by recognizing that and can be combined into three-plane aggregates (the subscript indicates the number of temporal planes):

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In addition, the constraints of our reference frame result in only four independent parameters to this metric formula, not six. With these constraints and substitutions, the metric formula for a three-dimensional projection of a six-dimensional manifold with a single evolution parameter is:

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Where is the distance between two points, and are the temporal coordinates of those points, and , , and , are the spatial distances.

# INitial Conditions

There are three initial conditions in Eq. (8): the constant acceleration, , the initial tangent velocity, , and the age of the manifold at the time of observation, .

If we chose a line-of-sign path along a null geodesic (that is, a path having and ) then the distance between two points can be found by solving Eq. (8) for :

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This formula can be simplified by encoding the time coordinates as:

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Where , the redshift, is the change in a unit length (wavelength) from to . This redshift value is encoded in photons, which we assume to travel along the null geodesic, making it possible to use a volume of photons from a known source (of luminosity) as proxies for luminous distance markers. As a function of redshift, the luminous distance is:

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Chart

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Figure 3 - Top: The luminous distances to a selection of 482 SNe Ia supernovae (green) and the distance predicted by the quadratically expandig space metric formula (red) and, for comparison, the distance predicted by the FLRW metric formula (blue)[[2]](#footnote-2). Bottom: The difference between the predicted luminous distance and the observed value in Gpc.

Using the combined data from (Conley et al. 2010), (Rodney et al. 2012), (Jones et al. 2013), (Rodney et al. 2016) and the parameters from (Rodney et al. 2016) to normalize the sets, we can extract the initial conditions using a chi-square minimization algorithm. The supernovae data demonstrates that this model is a better match to the observed universe than FLRW, with fewer free parameters, and provides us with a fiduciary model with which we can continue our discussion.

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Table 1– The initial conditions of the fiduciary model.

# Motion

A particle on the surface of this manifold in free fall will accelerate. The equation describing this motion can be derived from Eq. **Error! Reference source not found.**:

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This can be expanded with the chain rule:

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Where is the proper acceleration of an object, and is the acceleration caused by a fictitious force (that is, the change in the basis vector with time). Our geodesic equation is:

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In non-relativistic domains, the equation for the motion of a free-falling particle with a mass of in the presence of a collection of fictitious forces, , is:

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Where is the momentum. We can derive a formula for orbital motion in a gravitational field by replacing the general terms of Eq. (14) with more specific terms.

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Where is the gravitational constant, is the mass within a radius, , and is the tangential velocity of an object at the given radius.

Chart, line chart

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Figure 4 –The velocity curves of orbital motion around a spherical mass of having a radius of 4 kpc and constant density, for our fiducial model in quadratically expanding space (red), and Newtonian dynamics (that is, ) (blue).

# Fundamental Plane

Eq. (15) defines relationship between the tangential velocity, the radius of orbiting objects and the enclosed mass. This formula can be rearranged to predict the maximum mass possible in gravitationally bound orbit.

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The radius where the maximum mass will be found is:

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Substituting the Eq. (17) back into Eq. (16) yields the formula for the maximum mass given the velocity:

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Chart, radar chart

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Figure 5 - The fundamental plane where the radius is in , the velocity is in , and the mass is in . Solid line is the maximum mass allowed for orbital motion in quadratically expanding space.

A study of the relation between velocity and mass was conducted in (McGaugh 2012). Employing a minimization algorithm on Eq. (18) and solving for , we find a value of , which we used earlier to define our fiducial model. The combined stellar and gas masses of a collection of gas-rich spiral galaxies are displayed in Figure 6 and overlaid with Eq. (18) using our fiducial model.

Chart, scatter chart

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Figure 6 - The relationship between tangential velocity and mass. The blue circles are the combined gas and stellar mass of the gas-rich galaxies and the solid line is the maximum mass allowed by quadratically expanding space for orbital motion.

# Conclusion

The assumption that objects at rest remain at rest is the root cause of every wrong prediction of General Relativity, and there are several. If we allow ourselves the conceit to critically examine a belief that we acquired when we still had a favorite blanket, then we find a perfectly good theory of gravity that agrees with observations on every scale without the need for new physics.

# Data Availibility

The data and Mathematica notebooks underlying this article are available at https://github.com/DonaldAirey/quadratically-expanding-space.

# Bibliography

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1. Corresponding author: shepard@airey.us [↑](#footnote-ref-1)
2. As per the parameters found in (Planck Collaboration et al. 2019) [↑](#footnote-ref-2)