The Dynamic Properties of a FOUR-DIMENSIONAL, Complex Manifold With A Uniform, Non-Zero Acceleration FIeld

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# Abstract

Friedman’s solutions to Einstein’s field equations do not make accurate predictions about the universe on the largest scales. Even with multiple free parameters, or perhaps because of them, General Relativity gives conflicting answers when asked for the rate of expansion of the universe. Here we offer an alternative model of expansion employing a four-dimensional, complex manifold with one imaginary dimension of time and three real dimensions of space and no other gimmicks. Like all manifolds, this one has an acceleration field. Unlike other manifolds, this field is not assumed to be zero. We then provide an evolution parameter, , and describe the resulting dynamic properties. Here we demonstrate that General Relativity, with a model of quadratic expansion, makes more accurate predictions on all scales than it does with an assumption of Lorentz symmetry. When given the proper metric tensor, General Relativity can even predict the Tully-Fisher Relationship. However, to do this, we must abandon one of our most sacred beliefs: that objects at rest remain at rest.

# Introduction

With every new revelation from our space telescopes, the *Theory of General Relativity* appears to be more broken. When the field equations are solved using the FLRW metric tensor, they give conflicting answers for the rate of expansion of the universe () that is beyond the ability of the experimenters to explain. It gives one answer when we look at background radiation, and a different answer when we look at supernovae. In the language of the Scientific Method, the Friedman solutions to the Einstein Field Equations have disproven themselves.

Perhaps there is more new physics to be discovered? General Relativity already depends on multiple ad hoc theories – essentially free parameters in the energy-momentum density term – that have no plausible theoretical foundation (e.g., Dark Energy) and the predictions still do not match the observations.

Perhaps an assumption was missed? Yes, perhaps. Shall we start with the first one? All forces cause acceleration. Not all acceleration is caused by forces. Some acceleration is caused by curvature. The curvature of coordinate time with respect to proper time, for example, would be observed as an acceleration without a force. It is illogical to assume that two particles would not drift apart when all the forces are removed. If the curvature is not known, then the acceleration is not known. What law allows us to assume that objects at rest remain at rest?

There is none. Objects in freefall will follow the surface of the manifold. That is the only law that matters. If that surface possesses an acceleration, then free-falling objects on that surface will also accelerate as described by:

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Where is the acceleration of an arbitrary point on the manifold, is the velocity, is the evolution parameter (proper time), and represents the basis vectors for the four dimensions – time (, radius (), polar angle (), azimuthal angle () – in this coordinate system where . This relation can be expanded with the chain rule:

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Where is the proper acceleration of an object, and is the acceleration from the basis vectors changing with respect to the coordinates. Rearranging, the formula for the proper acceleration of a test particle on the surface of this manifold having a non-zero acceleration field, , is:

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# Galaxy Rotation Curves

In non-relativistic domains, the equation describing the path of this test particle in the presence of a collection of fictitious forces, , is:

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Where is the momentum. We can derive a formula for motion by replacing the general terms of this equation with specific terms for centripetal motion in a gravitational pseudo-force field:

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Where is the gravitational constant, is the mass within a radius, , and is the tangential velocity of an object at the given radius. What, then, would freefall motion look like on the surface of complex manifold with an arbitrary acceleration field?

A graph of a graph

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Figure 1 - The rotation curves of orbital motion around a spherical mass of having a radius of and constant density, for a variety of freefall accelerations (in units of ).

From Figure 1 we observe that the greater the acceleration field of the manifold, the faster an object in that field will orbit a given mass. This is an easily identified property that sets quadratic expansion apart from flat spacetime. Note that at higher values of , we see a flattening of the rotation curves on a galactic scale, and at , we recover Newtonian physics.

# The Tully-FisheR Relation

Orbital systems in a uniform acceleration field possess a plane of mass that is a function of radius and tangential velocity. Eq. (2) can be rearranged to predict the enclosed mass in such a system.

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A graph of a graph showing a curve

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Figure 2 - The fundamental plane of mass, in units of , for , as a function of radius, in , and velocity, in .

This relationship suggests that there is a radius beyond which the acceleration field outward overcomes the gravitational field inward, as seen in Figure 2, placing an upper bound on the amount of mass that can be enclosed in an orbit. The radius of this inflection point is:

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We can substitute Eq. (5) back into Eq. (4) and find the formula for the maximum mass, , as a function of tangential velocity:

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A study of the relationship between velocity and baryonic mass was conducted in (McGaugh 2012). Using the data from this study and employing a minimization algorithm on Eq. (6), we find a value of for , thus:

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Where velocity has units of and mass has units of .

Blue dots on a black background

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Figure 3 - The relationship between tangential velocity and baryonic mass. The blue circles are the combined gas and stellar mass of the gas-rich galaxies and the solid line is the maximum mass possible in quadratically expanding space for the given velocity.

Eq. (7) predicts the Tully-Fisher relation with one free parameter. Incontrovertible evidence of an acceleration field is written in the night sky. Quadratic expansion predicts that galaxies will have a brightness that is limited by the fourth power of their tangential velocity. This is another easily identified property of a uniform acceleration field. A metric employing Lorentz symmetry does not predict such a relationship.

# The Metric Formula

The distance between two points, , on this manifold is described by:

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where is the metric tensor, and is the difference between two coordinates in each dimension. If we consider only time and radius, and assume the coordinate axes are orthogonal, this formula expands to:

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where , is the change in proper distance and is the change in proper time. As observers, we can only measure coordinate distances, so a conversion is required. The coordinate distances are equal to the second integral of the acceleration field of the manifold.

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Where is the tangent velocity of the manifold. Eq. (11) describes uniform acceleration on a manifold where is the initial tangent velocity and is the change in coordinates for the given dimension for a given interval of to .

A black background with purple and white letters

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Figure 4 - The interval, , from to showing the line elements: proper space () and proper time ().

First, we need a relation between proper time and coordinate time. Figure 4 illustrates that, on an expanding manifold, a change in proper time is equal to the change in coordinate time as . Remembering that time is imaginary (units of ), we have:

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Next, we need the relation of coordinate space to proper space. We start by describing the relationship between and as a ratio of wavelengths as the manifold expands.

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Where is a unit length at and is a unit length at . The proper distance, , is the coordinate distance, , minus one half of expansion (expressed as ):

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The metric formula for a two-dimensional distance using practical coordinates is thus:

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# Type Ia Supernovae

There are three initial conditions in Eq. (16): the acceleration, the initial tangent velocity, and the age of the manifold. The uniform acceleration, ­ has already been determined from the Tully-Fisher relation. The initial tangent velocity, , can be derived from the present-day speed of light. We assume that light travels along a null geodesic () at the same speed as everything else on the manifold, specifically:

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On the surface of this manifold, information travels at the velocity of time. The last free parameter, the age of the universe, can be calculated using photons from a special kind of supernovae, type Ia.

The distance to an object can be found by solving Eq. (16) for :

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This can be simplified by encoding the time coordinates as:

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Where is the radius as a function of redshift and is the redshift. This redshift value is encoded in photons making it possible to use a volume of photons from a known source of luminosity as distance markers. As a function of redshift, the luminous distance, , is:

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A screenshot of a computer screen

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Figure 5 - Top: The luminous distances to a selection of 1071 Ia supernovae (green) and the distance predicted by luminous distance formula in Eq. (18) (red) and, for comparison, the distance predicted by the FLRW formula (blue)[[2]](#footnote-2). Bottom: The difference between the predicted luminous distance and the observed value in Gpc.

Using the data from the Pantheon data set, we can extract the initial conditions using a chi-square minimization algorithm. The reduced of for uniform acceleration compared to for FLRW demonstrates that this model is a better match to the observed universe, with just one free parameter.

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Table 1 – The initial conditions: the uniform acceleration, the initial tangent velocity, and the age of the manifold.

The SNe Ia data describes a manifold that is 13.3 billion years old.

# Data Availibility

The data and Mathematica notebooks underlying this article are available at https://github.com/DonaldAirey/quadratically-expanding-space.

# Bibliography

Conley, A., Guy, J., Sullivan, M., et al. 2010, Astrophys J Suppl Ser, 192 (IOP Publishing), 1

Jones, D. O., Rodney, S. A., Riess, A. G., et al. 2013, ArXiv Prepr ArXiv13040768

McGaugh, S. S. 2012, Astron J, 143 (IOP Publishing), 40

Planck Collaboration, Aghanim, N., Akrami, Y., et al. 2019, ArXiv180706209 Astro-Ph, http://arxiv.org/abs/1807.06209

Rodney, S. A., Riess, A. G., Dahlen, T., et al. 2012, Astrophys J, 746 (IOP Publishing), 5

Rodney, S. A., Riess, A. G., Scolnic, D. M., et al. 2016, Astron J, 151, 47

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2. As per the parameters found in (Planck Collaboration et al. 2019) [↑](#footnote-ref-2)